AUTOMATIC MESH REFINEMENT IN GLOBAL DIGITAL IMAGE CORRELATION

Wittevrongel L., Debruyne D., Lava P. and Lomov S.V.
Department of Material Engineering, University of Leuven Campus Ghent
Gebroeders De smetstraat, Belgium
Lukas.wittevrongel@kuleuven.be

1. INTRODUCTION

Digital Image Correlation (DIC) is widely used in the field of experimental mechanics due to its low experimental cost such as simple setup, specimen preparation and low requirements in measurement environment. The traditional approach for DIC is a subset based approach, where pixels are individually tracked using its neighboring pixels [1]. The lack of inter-subset continuity increases the sensitivity of the local approach to noise. Eventually, the measured displacements are smoothed prior to differentiation to minimize the effect of non-continuity on the strain. The downside of this method is that the choice of subset, step and strain window size influences the spatial resolution and thus makes data very user dependent [2]. To avoid the non-continuous displacement field, the alternative global approach is developed [3]. By tracking a full mesh, and thus all pixels at once from reference to deformed image, a continuous displacement field is obtained. By using a global approach, the problem of continuity is solved but the user dependency still remains. All global approaches use a mesh with fixed element orders and thus a “converged” element size should be chosen. Using an element with a certain polynomial order (Q4, Q8, 24-node element) and a certain element size, inevitably leads to natural smoothing and consequentially to discretization errors [4]. A possible solution to this problem is using h-refinement. In this refinement, the element size is reduced in regions where the deformation is more complex and thus where discretization errors are large. The fact that this refinement is done by the experimenter and is mostly based on pre-knowledge of the solution still leads to very user/approach dependent results. To circumvent the user dependency of the results, a self-adapting global DIC procedure is proposed. Instead of refining the mesh based on pre-knowledge, the mesh is automatically refined using convergence procedures [5].

2. HIERARCHICAL GLOBAL DIC

The most common techniques to achieve mesh refinement are h-, r- and p-refinement. Using the h-method, used in current global approaches, mesh refinement is obtained by reducing the element size while maintaining the element order. The downside of h-refinement is that in global DIC the decrease of element size results in a proportional decrease in included pixels. A decrease in pixels results in less information and included features causing correlation issues. Further, re-meshing of the ROI is needed. Another approach is r-refinement, where the nodes are moved to obtain a more appropriate mesh. Here, the downside is that the total DOF within the mesh is fixed and adaptivity is limited by the initial mesh. In the proposed algorithm, mesh refinement is obtained by upgrading the elements to a higher polynomial order and thus adding extra DOF to the elements themselves. This approach is also known as p-refinement. The extra DOF ensure that complex deformations can be represented while the element size is not reduced and thus no re-meshing is performed. Further it will be shown that using p-elements can lead to an automatic refinement procedure.

A novel global DIC algorithm that uses adaptive p-elements is p-DIC [5]. The approach is based on conservation of optical flow, similar as most global DIC approaches. The displacement field occurring between reference image f and deformed image g can be found by minimizing the cost function ϵ (1) with respect to the displacement parameters $d_{i\alpha}$ used in the displacement description d (2) [6]:

$$\epsilon^2 = \iint_{\Omega e} \left(f(\boldsymbol{x}+d) - g(\boldsymbol{x}+d')\right)^2 dx \tag{1}$$

$$d = \sum_i \sum_\alpha \Phi_i(\boldsymbol{x}) d_{i\alpha} \tag{2}$$

Where d represent the displacement described by the shape functions Φ_i (x) and displacement parameters $d_{i\alpha}$. The choice of shape functions defines the number of DOF’s *i,* and thus the order/polynomial degree of the elements used (e.g. Q4, Q8 elements). Other than in the current approaches, the shape functions are not fixed in the p-DIC approach. By using hierarchical shape functions a generic displacement description of order p can be obtained for the considered element [7]. Updating an element from order p to order p+1 is obtained by simply adding some extra hierarchical shape functions to description (2) without altering the lower order shape functions already in the element. The extra functions can be assigned to both the element edges as the element surface. Hierarchical shape function until 8th order are shown in Fig 1.

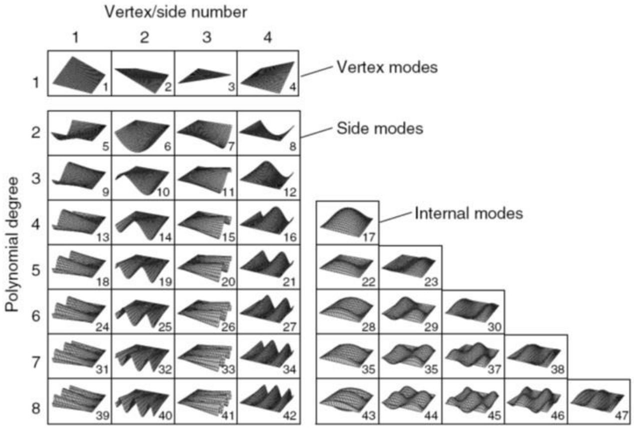

Figure 1 – Graphical representation of hierarchical shape functions [9]

Using this approach, element orders can be adapted during the correlation. Even more, elements can receive different orders within the mesh to represent different complexities of deformations. Because only DOF are added to the displacement description, the element geometry is not altered and therefore the element size is not reduced. Keeping the element large ensures that no correlation issues occur and that the elements remain robust to noise [8]. For these reasons the p-refinement can be an attractive refinement scheme to be used in DIC, despite the more complex calculations and code.

A validation of this approach has been performed in a previous study, confirming the concept of hierarchical higher order elements in a global DIC algorithm [5]. To recapitulate, it was shown that for homogeneous applications the performance in terms of spatial and displacement resolution the p-DIC and local subset method where competitive. For heterogeneous applications though, the p-DIC method has a considerably large gain in displacement resolution for the same spatial resolution. Furthermore, the extra advantage that for the p-DIC method results are considerably less user dependent is proven. And thus, the p-DIC method is favorable for measuring high gradients with small displacement signals.

3. AUTOMATIC REFINEMENT

In order to fully benefit from the adaptive mesh introduced above, refinement should be automatic. In FEA, refinement of the mesh is often driven by convergence of the element energy. The refinement procedure of the finite element mesh (introducing extra DOF) continues until the element energy for all elements is converged. In p-DIC method, a similar approach is followed. Since in p-DIC no material parameters are known, not the exact strain energy but an L2-norm is used as it is widely known that both norms are equivalent. The used L2-norm for element *e* in strain can be written as:

$$E_\varepsilon^e = \iint_{\Omega_e} \varepsilon^T \varepsilon \, d\Omega \tag{3}$$

In the p-DIC method, an element keeps being refined until its L2-norm converges. For the mesh, refinement continues until all elements are converged. In this way each element has its own most optimal polynomial order for its own underlying deformation field. When reaching a certain max element order (e.g. 12^{th} order) and not all elements obtain convergence in strain L2-norm, the correlation is restarted and the displacement field is used in the convergence procedure. The previous is proposed because in a finite element mesh the displacements have a convergence rate one order higher than the strains. When the elements even do not convergence in the displacement norm, the final option is to use global norms. Here, a global strain or displacement norm over the hole ROI is used to control the mesh refinement. It should be noted that when using the global norms all the elements contain the same global convergence curve, and thus the algorithm will only perform uniform refinement. A global convergence procedure improves stability, but has less performance as the elements are not optimized individually. In table 1, the hierarchy in L2-norms are shown.

	Norm	Type	Energy area	Refinement
1	$E_\varepsilon^e = \iint_{\Omega_e} \varepsilon^T \varepsilon \, d\Omega$	Local	Element	Element by element
2	$E_\varepsilon^e = \iint_{\Omega_e} u^T u \, d\Omega$	Local	Element	Element by element
3	$E_\varepsilon^e = \iint_{ROI} \varepsilon^T \varepsilon \, d\Omega$	Global	ROI	Uniform
4	$E_\varepsilon^e = \iint_{ROI} u^T u \, d\Omega$	Global	ROI	Uniform

Table 1 – Hierarchy of L2-norms

4. IMPLEMENTED APPROACH

In Figure 2, the flowchart of p-DIC is shown. It is seen that the only user input is discretising the region of interest (ROI), defining the initial first order mesh. From there on the automatic procedure controls the mesh refinement, until convergence is found. When for all elements convergence is found a non-uniform higher order mesh is obtained, where each element has its most optimal polynomial order.

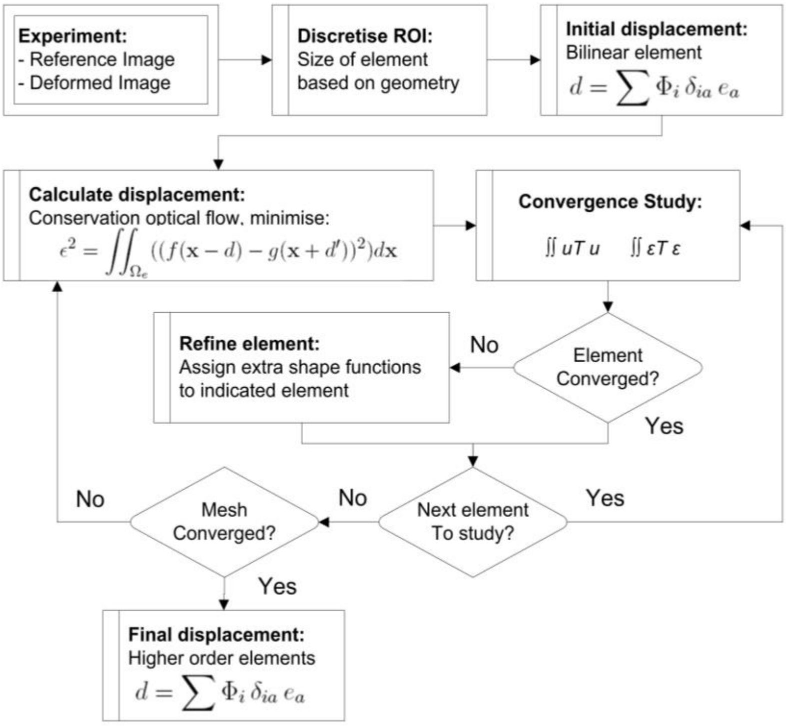

Figure 2 – Flowchart of p-DIC, an automatic refined higher order global DIC algorithm.

Figure 3 indicates an example of the automatic refinement in the p-DIC approach. A vertical tensile test on a perforated specimen is performed. In Figure 3 the reference mesh and tracked mesh are shown on the left. On the right the final element order distribution and the convergence curves for each element of the mesh are shown.

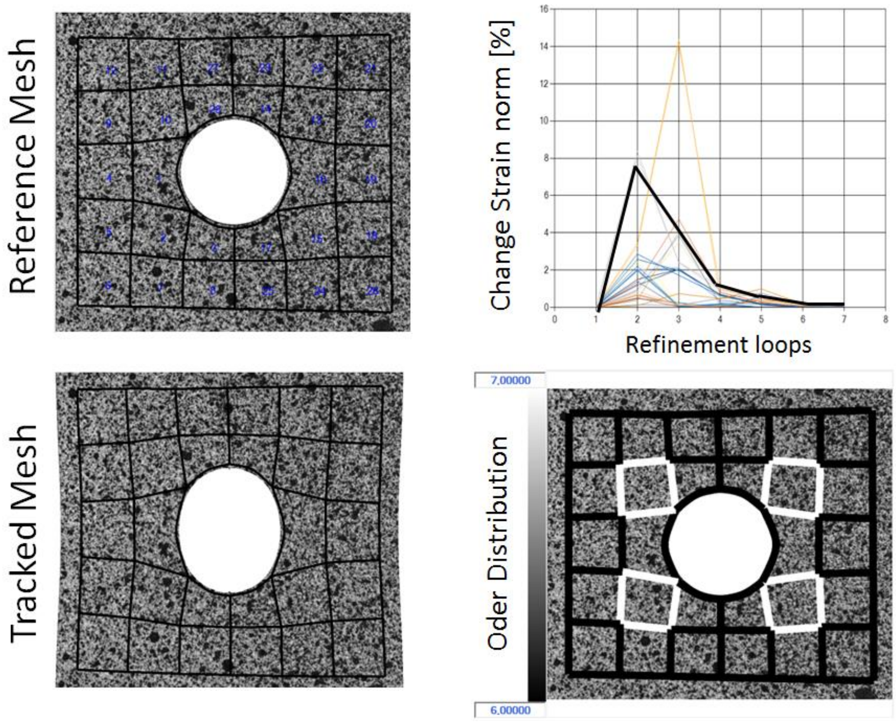

Figure 3 – Illustration of the p-DIC method by a vertical tensile test on a perforated specimen.

5. VALIDATION

The algorithm using the higher order elements is validated and compared to the local subset method. In the validation both numerical as practical experiments are used. Also, both displacement and strain resolution with their corresponding spatial resolutions are investigated. Results indicate that p-DIC and the subset method have similar performance. In homogeneous applications the performance in terms of spatial and displacement resolution the p-DIC and local subset method are competitive. For heterogeneous applications though, the p-DIC method has a considerably large gain in displacement resolution for the same spatial resolution. In a second validation, the performance of the refinement procedure is checked. Here also both numerical as experimental images are used. To summarize, it is shown that convergence in L2-norms is a valid tool for mesh refinement in global DIC. The element size does not influence the final results, but only influences the polynomial order of the elements themselves. As second, the robustness to experimental influences is investigated. It is showed that the refinement procedure is able to cope with varying noise and light conditions as well as with different correlation criteria.

6. CONCLUSION

A full automatic refined global DIC algorithm is presented. It is based on the combination of a novel adaptive global DIC algorithm containing higher order p-elements and convergence of element energy (equivalent L2 norm). Both principles are combined in the correlation platform 'AdaptID', housing the automatically refined global DIC approach. Using the global approach insures a continuous displacement field while using higher order elements result in large elements with a good spatial resolution and noise robustness. The implementation of hierarchical p-elements leads to the possibility of adapting element during correlation. Automatic refinement is obtained by using convergence in element strain/displacement norms. In the proposed method, an element keeps being refined until its L2-norm (strain or displacement) converges. For the mesh, refinement continues until all elements are converged. In this way each element has its own most optimal polynomial order. Validation of the proposed algorithm showed great performance in terms of resolutions and robustness to experimental influences. An extra major benefit is the automatic mesh refinement which rules out most user dependency.

REFERENCES

1. M Sutton, J Orteu, and H Schreier (2009). Image Correlation for Shape, Motion and Deformation Measurements. *Springer*

2. P Lava, S Cooreman, S Coppieters, M De Strycker, and D Debruyne (2009) Assessment of measuring errors in dic using deformation fields generated by plastic fea. *Opt. Las. Eng., 47:747-753.*

3. G Besnard, Hild, and S Roux (2006) "finite-element" displacement field analysis from digital images: Application to Portevin–Le Châtelier Bands. *Experimental Mechanics*, 46:789-803

4. I Babuska, J.R. Whiteman, and T Strouboulis. (2011) Finite Elements. An introduction to the method and error estimation. *Oxford university press*

5. Wittevrongel L, Debruyne D., Lava P and Lomov S.V. (2014) A self adaptive global digital image correlation algorithm. *Experimental Mechanics, DOI 10.1007/s11340-014-9946-3*

6. O.C Zienkiewicz and R.L Taylor (2000) The finite element method. Volume 1: The basis. *ISBN 0 7506 5049 4*

7. B Szabo, A. Duster, and E Rank. (2004) The p-version of the finite element method. *Encyclopedia of Computational Mechanics* ISBN: 0-470-84699-2

8. F Hild and S Roux (2011). Comparison of local and global approaches to digital image correlation. *Experimental Mechanics, 52:1503-1519*

9. B Szabo and I Babuska (1991). Finite element Analysis. *Wiley*